ANSWERS AND EXPLANATIONS

Basic Operations

1. In order for 12 to be the result of this equation, you must divide 108 by 9. Insert the ÷ symbol in the blank.
2. To reach an answer of 3.5, you must divide 7 by 2. Insert the ÷ symbol in the blank.
3. One way to solve this problem is to look for the Lowest Common Denominator (LCD). The smallest number that both 4 and 8 divide evenly into is 8, so the fraction $\frac{3}{8}$ does not need to be changed. The fraction $\frac{1}{4}$ is equivalent to $\frac{2}{8}$, $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$, so insert the + symbol in the blank.
4. The Greatest Common Factor (GCF) is the largest number that divides evenly into any two or more numbers. List the factors of 48 and 72, then select the largest factor that they have in common:

48	72
1×48	1×72
2×24	2×36
3×16	3×24
4×12	4×18
6×8	6×12
	8×9

Based on this list, the GCF is 24.

5. The LCD is the smallest number into which all of the denominators will divide evenly. For this problem, you must find the smallest number into which 8 and 4 will divide evenly. Since 4 will divide evenly into 8 ($\frac{8}{4} = 2$), 8 is your LCD. You can now change $\frac{3}{4}$ to $\frac{6}{8}$ by multiplying both the numerator and denominator by 2 (the amount of times 4 goes into 8).
6. You must first complete the mathematics within the parentheses $(96 - 21 = 75.)$ Next, do any multiplication or division in the problem, from left to right. Here, you have 75 divided by 15, which equals 5. Finally, do any addition or subtraction in the problem, from left to right: 5 plus 11 gives us an answer of 16.
7. You must first do the operations within the parentheses $(27 + 2 - 3 = 26.)$ Now multiply the value from the parentheses by 3: 3 times $26 = 78$.
8. You must first find the LCD for the two fractions involved. The denominators are 3 and 7. The smallest number into which both of these can divide evenly is 21. Convert each denominator to 21 by multiplying $\frac{1}{3}$ by $\frac{7}{7}$ and $\frac{3}{7}$ by $\frac{3}{3}$. This gives you $\frac{7}{21} + \frac{9}{21}$, which equals $\frac{16}{21}$.

9. This is a simple subtraction problem. To solve this without a calculator, line up the decimal points and subtract, remembering to "borrow" and "carry," as follows:

$$\begin{array}{r} 231.2 \\ -\underline{198.7} \\ 32.5 \end{array}$$

10. First convert $\frac{1}{5}$ to a decimal, which is 0.2. Then multiply 0.25 by 0.2, which gives you an answer of 0.05. Another way to solve this is to first convert 0.25 to a fraction, which is $\frac{1}{4}$. When multiplying the two fractions, you first multiply the numerators, and then the denominators, giving you $\frac{1}{20}$. Because this is equivalent to 0.05, either answer will be correct.

Square Roots

1. 5^2 simply means 5 times 5, which equals 25.
2. Find the square roots before you do the division. The square root of 36 is 6, and the square root of 4 is 2. Next divide 6 by 2, which equals 3.
3. "3 times 3" can be stated as "3 squared." The proper way to write this is 3^2.
4. Both numbers are raised to the power of 2 (they are squared). You must first find these squares before you do your subtraction. 7 squared is 49, and 3 squared is 9. So, your answer is $49 - 9$, which equals 40.
5. This problem requires you to find a square root of a number as well as a number squared. The square root of 64 is 8, and 2 squared equals 4. Your answer is 8 times 4, which is 32.

Properties of Integer Exponents

1. According to the rule $x^m \times x^n = x^{(m-n)}$; therefore, add the exponents together. $x^3 \times x^6$ is equal to x^{3+6}, or x^9.
2. A rule regarding exponents states that $(x^m)^n = x^{mn}$. Applying this rule gives you $(3^2)^3$, which yields 3^6. 3 to the 6th power is 729.
3. The exponent is distributed to both the numerator and the denominator, creating $\frac{5^3}{3^3}$, or $\frac{125}{27}$.
4. The answer to this problem is 1. For any value x where $x \neq 0$, $x^0 = 1$.
5. One of the rules regarding exponents tells you that $(xy)^m = x^m \times y^m$. Applying the rule gives you the following:

   $y^2 \times z^2$, or y^2z^2

Exponents

1. The power that a number is raised to is equivalent to the number of times you multiply that number by itself: $2 \times 2 \times 2$ is equal to 8, so the answer is 2 raised to the power of 3 (2^3).
2. 3^3, or 3 to the 3^{rd} power, means you must multiply $3 \times 3 \times 3$, which equals 27.
3. You must find a number that, when raised to the power of 4, equals 81. Because 81 is a perfect square (9×9, or $9^2 = 81$), and 9 is a perfect square (3×3, or $3^2 = 9$), you can simply square 3^2 to arrive at 81: $(3^2)^2 = 3^4$.
4. $5^3 = 5 \times 5 \times 5$, which gives you 125.

5. When raising an exponent to another power, multiply the exponents ($4 \times 2 = 8$). So, the answer is 2^8, or 256.

Scientific Notation

1. When dealing with scientific notation, the power of 10 indicates the number of spaces you must move the decimal place, either to the right (for a positive value), or to the left (for a negative value.) To turn 4.237 into 423,700,000, you must move the decimal place 8 spaces to the right. Therefore, 10 needs to be raised to the power of 8 (10^8).
2. To solve this problem, you must simply move the decimal point the number of times indicated by the power of 10. Since you are given 10^5, you know that you must move the decimal point 5 spaces, to the right because the exponent is a positive number. This gives you an answer of 376,000.
3. This problem can be set up as $\left(\frac{2.50}{1.25}\right) \times \left(\frac{10^4}{10^3}\right)$. The first half $\left(\frac{2.50}{1.25}\right)$ gives you 2. When dividing like bases, you subtract your exponents ($4 - 3 = 1$). You are left with 2×10^1. Since 10 to the 1st power is 10, the multiplication leaves you with an answer of 20.
4. You are given a negative value for the power to which 10 is raised (-5). This means that you must move the decimal point 5 spaces to the left to get your answer, which is .0000647.
5. You can set this problem up as $(4.2 \times 1.8) \times (10^3 \times 10^{-6})$. The first half of the equation (4.2×1.8) gives you 7.56. When multiplying like bases, you add your exponents: $3 + (-6) = -3$. Therefore, you are left with 7.56×10^{-3}, which can be expressed as 0.00756.

Ratio, Proportion, and Percent

1. To solve this problem, you can set up a proportion. You are looking for a number that is 30% of 20. The proportion looks like $\frac{x}{20} = \frac{30}{100}$, because the unknown number is equivalent to 30 out of the 100 parts of the whole (20). To solve, you cross-multiply, leaving you with $100x = 600$. Divide both sides by 100: $x = 6$.

2. You are given a proportion to solve. To find the answer, cross-multiply, giving you $78x = 234$. Dividing both sides by 78 will give you the answer $x = 3$.

3. To answer this question you must determine the ratio of organisms to liter of river water. The problem states that a 2-liter sample of water contained about 24 organisms, and a 4-liter sample of water contained about 48 organisms. Upon closer examination of this information you will see that the ratio of organism, to water is the same in each sample. Therefore, you can set up a ratio using one sample:

   2 liters of water yields 24 organisms.

   This can be expressed as 2 to 24, or 2:24, which can be reduced to 1:12. For every 1 liter of water you will see 12 organisms. Therefore, 10 liters of water will contain 120 organisms.

4. You need to set up a proportion. You are given that 20% of x is equal to 16, and you want to find the value of x. The proportion looked like this:

   $$\frac{16}{x} = \frac{20}{100}$$

After cross-multiplying, you are left with $20x = 1{,}600$. After dividing both sides by 20, you have the answer: $x = 80$.

5. Once again, you need to use a proportion to solve this problem. You know that Jim scored 95 points in 5 games, and you want to find out how many points he will score in a total of 12 games. Your proportion will look like this:

$$\frac{95}{5} = \frac{x}{12}$$

Cross-multiplying will leave you with $5x = 1{,}140$. Divide both sides by 5, and you get your answer, $x = 228$. If Jim continues to score at this rate, he will score a total of 228 points by the end of the season (12 games).

Linear Equations with One Variable

1. First isolate the unknown number (the variable) on one side. To do this, you add 17 to both sides, giving you $3x = 63$. Next, you divide both sides by 3 to get the x alone. This gives you the answer: $x = 21$.
2. Multiply both sides by 4 to get rid of the fraction and leave the x on its own. This gives you $x = -24$.
3. You are given the value of x, and you are looking for a missing number in the equation. If $x = 15$, then $4x = 60$. So you are left with the equation $60 - (\text{some number}) = 42$. Subtract 60 from both sides to get 18.
4. This is a standard Rate × Time = Distance problem. Since the two trains start 600 miles apart, you know that their combined distance traveled must equal 600. Using the R × T = D formula, you can say that (Rate of Train 1 × Time of Train 1) + (Rate of Train 2 × Time of Train 2) = 600. You know how fast the trains are moving, and their total distance, but you do not know the time, so solve for T. Train 1 travels at 90 mph for T hours, while Train 2 travels at 75 mph for T hours. Your equation will look like this:

$$90T + 75T = 600$$
$$165T = 600$$
$$T = 3.64 \text{ hours}$$

5. First do the multiplication on the left side of the equation. This gives you $3x - 12 = 5x - 20$. Next, you need to group the like terms together. To do this, subtract $3x$ from both sides, and add 20 to both sides. This leaves you with $8 = 2x$. Dividing both sides by 2 will give you the answer: $x = 4$.

Absolute Value

1. First do the subtraction within the absolute value lines, $(-8 - 6 = -14)$. Absolute value is the numerical value of a real number without regard to its sign. Therefore, the absolute value of -14 is 14.
2. To solve this problem, you need to set up two equations: $4x - 6 = 10$, and $4x - 6 = -10$. You then solve both for x.

$$4x = 16, \text{ and } 4x = -4$$
$$x = 4, \text{ and } x = -1$$

3. In order to perform the multiplication in this problem, you must first find the absolute value of both numbers. The absolute values of -15 and 6 are 15 and 6, respectively. The answer is 15×6, which equals 90.
4. To find the possible answers for x in this problem, you must set up two equations:

   $6x+8=3x-7$, and $6x+8=-(3x-7)$.

   First, you need to distribute the minus sign in the second equation, giving you $6x+8=-3x+7$.

   You then solve both for x:

   $3x=-15$, and $9x=-1$
   $x=-5$, and $x=-\frac{1}{9}$
5. First find the absolute value of the denominator. The absolute value of -8 is 8. Now you can perform the division. -32 divided by 8 gives you an answer of -4.

Simple Probability

1. On a 6-sided die, there are 3 even and 3 odd numbers. Therefore, the probability that you will roll an odd number is 3 out of 6, or $\frac{3}{6}$. This can be reduced to $\frac{1}{2}$, or .5
2. If 2.5% of the CD players produced by this company are defective, then the number of defective devices out of 300,000 can be determined by multiplication $0.025 \times 300{,}000 = 7{,}500$.
3. When flipping a coin, there are only two possible outcomes: heads or tails. Therefore, each side has a probability of $\frac{1}{2}$, or .5, of landing facing up. The chances of the coin landing on tails four times in a row can be expressed as $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$, or $\left(\frac{1}{2}\right)^4$. The final answer is $\frac{1}{16}$.
4. In this question, you can look at probability as a percentage. The probability that Dave will go to class is 0.7, or 70%. Therefore, the probability that he will NOT go to class is 100% – 70%, or 30%, which is equivalent to 0.3. Either answer is correct.
5. There are a total of 20 marbles in the bowl, 6 of which are red. If one marble is selected at random, the probability that it will be red is $\frac{6}{20}$ (the # of red marbles/the total # of marbles.) This can be reduced to $\frac{3}{10}$.

Functions

1. To solve, substitute 6 for x in the function:

   $f(6)=6^2-4(6)+8$
   $f(6)=36-24+8$
   $f(6)=20$
2. To solve, substitute $(x+1)$ for x in the function:

   $f(x+1)=(x+1)^2$
   $(x+1)(x+1)$
   $x^2+x+x+1$
   x^2+2x+1

3. The problem gives $g(x)=3x$ and $f(x)=x+2$ and asks for $g(f(x))$. The function $g(f(x))$ means that all of the x values in $g(x)$ are replaced with $f(x)$, as follows:

   $g(f(x))=3(f(x))$
   $g(f(x))=3(x+2)$
   $g(f(x))=3x+6$

4. To solve, substitute 2 for x in the function:

   $f(2)=2^4-\frac{3(2)}{2}$

   $f(2)=16-\frac{6}{2}$

   $f(2)=16-3$

   $f(2)=13$

5. To solve, substitute -5 for x in the function:

   $f(-5)=(-5)^2+(-5)$
   $f(-5)=25-5$
   $f(-5)=20$

Polynomial Operations and Factoring Simple Quadratic Equations

1. To solve the equation, substitute 4 for x:

   $3(4^2)-5(4)+9$
   $3(16)-20+9$
   $48-20+9=37$

2. To add or subtract polynomials, combine like terms (remember to keep track of the negative signs!):

   $(5x^3+3x-12)-(2x^3-6x+17)$
   $(5x^3-2x^3)+(3x+6x)-(17-12)$
   $3x^3+9x-29$

3. Use the distributive property to multiply each term of one polynomial by each term of the other (remember to use the FOIL method).

   $(4x^2+2x)(x-6)$
   First terms: $(4x^2)(x)=4x^3$
   Outside terms: $(4x^2)(-6)=-24x^2$
   Inside terms: $(2x)(x)=2x^2$
   Last terms: $(2x)(-6)=-12x$

   Now place the terms in decreasing order:

   $4x^3-24x^2+2x^2-12x$
   $4x^3-22x^2-12x$

4. Find two numbers whose product is -48 and sum is 2. The only possible numbers are 8 and -6. Therefore, the solution sets are $(x-6)$ and $(x+8)$.

5. The solution sets are given, so multiply the two sets together to find the original equation, using the FOIL method:

   $(x-4)(2x+3)$
   $2x^2+3x-8x-12$
   $2x^2-5x-12$

Linear Inequalities with One Variable

1. The inequality states that x must be greater than or equal to -5 AND less than 15. Therefore, x could be any number equal to or greater than -5, and also less than 15.
2. Solve this problem algebraically, as follows:

   $6x - 4x > 5 - (-3)$
   $2x > 8$
   $x > 4$

   x must be greater than 4 for this inequality to be true.
3. The value of x is given, so substitute 7 for x and calculate the value of both sides:

   $3(7) + 7 = 28$ and $5(7) - 6 = 29$

   The less than sign ($<$) is used because 28 is less than 29.
4. Once again, the first step in solving this problem is isolating the variable on one side of the inequality:

   $-5 - 20 < -3x - 2x$
   $-25 < -5x$
   $5 > x$

   It is important to remember that when dealing with inequalities, multiplying or dividing by a negative number involves reversing the sign. In this case, both sides were divided by -5, so the sign changes from $<$ to $>$.
5. To solve this problem, subtract 3 from both sides of the inequality:

   $-4 - 3 \leq x < 18 - 3$
   $-7 \leq x < 15$

   x is greater than or equal to -7 and it is less than 15.

Quadratic Formula

1. The quadratic formula is $x = -b \pm \dfrac{\sqrt{(b^2 - 4ac)}}{2a}$

   The first step in solving this problem is to substitute the numbers from the equation into the quadratic formula (keep in mind that the equation is in the form of $ax^2 + bx + c$).

   $$x = \frac{-22 \pm \sqrt{(22^2) - 4(10)(12.1)}}{2(10)}$$

   Next, simplify the problem to find the value of 22^2, which is 484.

   $$x = \frac{-22 \pm \sqrt{(484) - 4(10)(12.1)}}{2(10)}$$

   Next, do the rest of the multiplication, as follows:

   $$x = \frac{-22 \pm \sqrt{484 - 484}}{20}$$

   The square root of $484 - 484$ is simply 0, so you can disregard it for the rest of the problem. You are left with:

   $x = \dfrac{-22}{20}$ which simplifies to $\dfrac{-11}{10}$.

   Because the $\pm$ does not give separate answers, there is only one answer to the problem:

   $x = -\dfrac{11}{10}$, or -1.1.

2. $(4x^2 - 7x + 3) - (10x^2 + x - 11) = 0$
   $4x^2 - 7x + 3 - 10x^2 - x + 11 = 0$
   $-6x^2 - 8x + 14 = 0$
   Multiply the entire equation by -1:
   $6x^2 + 8x - 14 = 0$
3. For this problem, $a = 4$, $b = 1$, and $c = -5$. Substitute these numbers into the quadratic formula to get:

   $$x = \frac{-1 \pm \sqrt{(1^2) - 4(4)(-5)}}{2(4)}$$

   $$x = \frac{-1 \pm \sqrt{81}}{8}$$

   The square root of 81 is 9, so you now have:

   $$x = \frac{-1 \pm 9}{8}$$

   Because of the ± sign, you have two possible answers. Find them by making two separate equations:

   $x = \frac{8}{8}$ and $x = -\frac{10}{8}$

   Simplifying these two answers, you have your solutions: $x = 1$ and $x = -\frac{5}{4}$.
4. The first thing you must do is rearrange the equation to fit the format $ax^2 + bx + c = 0$. After doing this, the equation will be $4x^2 + 18x - 117$. Therefore, the values for *a*, *b*, and *c* respectively, are 4, 18, and -117.
5. First, use FOIL to create a trinomial equation.

   $(2x + 4)^2 = 0$

   $(2x + 4)\ (2x + 4) = 0$

   $4x^2 + 8x + 8x + 16 = 0$

   $4x^2 + 16x + 16 = 0$

   Now use $a = 4$, $b = 16$, and $c = 16$ in the quadratic formula, as follows:

   $$x = \frac{-16 \pm \sqrt{(16^2) - 4(4)(16)}}{2(4)}$$

   $$x = \frac{-16 \pm \sqrt{(16^2) - (16)(16)}}{8}$$

   $$x = \frac{-16 \pm \sqrt{0}}{8}$$

   $$x = \frac{-16}{8}$$

   $x = -2$

Radical and Rational Expressions

1. In this problem, you are dealing with radicals. When it comes to radicals, an important rule to remember is that $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$. Applying that

rule to this question, you see that $\sqrt{12} \times \sqrt{3} = \sqrt{36}$. The square root of 36 is 6.

2. By rule, $\sqrt{(a/b)} = \sqrt{a}/\sqrt{b}$. Therefore, $\sqrt{(2/5)} = \sqrt{2}/\sqrt{5}$. Eliminate the radical in the denominator by multiplying the quantity by itself and repeating this multiplication on the numerator:

$$\left(\frac{\sqrt{2}}{\sqrt{5}}\right) \times \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$$
$$= \frac{\sqrt{2} \times \sqrt{5}}{5}$$
$$= \frac{\sqrt{2 \times 5}}{5}$$
$$= \frac{\sqrt{10}}{5}$$

3. This question shows what is called a "cube root." The cube root of a number, x, is the number which raised to the third power gives x. This problem asks you to find the cube root of 27. Since $3 \times 3 \times 3$ is equal to 27, the cube root of 27 is 3.

4. To answer this question, you must first multiply the two parts of the equation, as follows:

$$\sqrt{2x^8} \times \sqrt{8y^2} = \sqrt{16x^4y^2}$$

You can simplify this in order to find the square root:

$$\sqrt{4^2(x^2)^2y^2}$$

Now that the problem is set up like this, the square root is clear:

$$\sqrt{4^2(x^2)^2y^2} = \sqrt{4^2} \times \sqrt{(x^2)^2} \times \sqrt{y^2}$$
$$= 4 \times x^2 \times y$$
$$= 4x^2y$$

5. The rule used in this problem is: $\sqrt[n]{a^m} = a^{(m/n)}$.
Therefore, $\sqrt[7]{11^{14}} = (11^{14})^{1/7} = 11^2 = 121$.

Inequalities and Absolute Value Equations

1. Since the inequality deals with an absolute value, $|7x - 13|$ will always be a positive number. For the inequality to be true, $7x - 13$ must be between the values of -22 AND 22. OR does not work here because the value must meet both the requirement of being larger than -22 as well as the requirement of being smaller than 22. If the absolute value is greater, use OR. If the absolute value is less than, use AND.
2. To solve this problem, you must first drop the absolute value sign, and then create two separate inequalities, in the form of $ax+b=c$. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c, as follows:

$$x+8 > 15 \qquad x+8 < -15$$
$$x > 7 \qquad x < -23$$

It is impossible for a value to be greater than 7 AND less than −23. Therefore, use OR.

$x > 7$ OR $x < -23$.

3. To solve this problem, you must drop the absolute value sign first, and then create two separate inequalities of the form $ax+b=c$. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c, as follows:

$2x+3 < 21$	$2x+3 > -21$
$2x < 18$	$2x > -24$
$x < 9$	$x > -12$

x must be less than 9 AND greater than −12. Unlike the previous problem, a number can meet both of these rules: $x < 9$ AND $x > -12$.

4. To solve this problem, you must drop the absolute value sign first, and then create two separate inequalities, of the form $ax+b=c$. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c. A value cannot be both greater than 29 and less than −29, so OR must be used. Set up the two inequalities to find that $5x-6 > 29$ OR $5x-6 < -29$.

5. To solve this problem, create two separate inequalities, as follows:

$-\frac{1}{4}x+3 > 5$	$-\frac{1}{4}x+3 < -5$
$-\frac{1}{4}x > 2$	$-\frac{1}{4}x < -8$
$x < -8$	$x > 32$

Because you multiplied both sides of each inequality by −4, you need to change the direction of the sign. Since x cannot be both less than −8 and greater than 32, OR is used: $x < -8$ OR $x > 32$.

Sequences

1. In order to solve this problem, it is crucial to know the formula for arithmetic sequences. This formula is $a_n = a_1 + (n-1)d$, where a_n is the particular term you are trying to find, a_1 is the first number in the sequence, and d is the common difference. This particular problem has already given you most of the information that you need. All that you have to do is substitute 3 for n, as you are looking for the 3rd term:

$$a_3 = 3 + (3-1)2$$
$$a_3 = 3 + (2)2$$
$$a_3 = 3 + 4$$
$$a_3 = 7$$

2. This question asks you to write your own formula for the sequence. You will need the first term in the sequence, as well as the common difference. The first number is −8, and noticing that you jump from −8, to −2, and then to 4, it is clear that the common difference is 6. Your formula should look like this:

$$a_n = -8 + (n-1)6$$

3. In this problem, you are dealing with a geometric sequence. These sequences have a formula that looks like this: $a_n = a_1(r)^{n-1}$. Here, r is the constant ratio. Looking at the sequence, it goes from $\frac{1}{4}$, to 1, to 4, and then to 16. This indicates that you must multiply by 4 each time;

therefore 4 is the constant ratio. To find the 6th term in this sequence, you must set up the following formula:

$$a_6 = \frac{1}{4}(4)^{6-1}$$
$$a_6 = \frac{1}{4}(4)^5$$
$$a_6 = \frac{1}{4}(1024)$$
$$a_6 = 256$$

4. First of all, you need to find an answer that is similar to the formula used for an arithmetic sequence: $a_n = a_1 + (n-1)d$. Looking at the choices, you can eliminate the second and third because they are formulas for a geometric sequence. In the sequence you are given, the first term is 7, and the common difference is 6. Therefore, the correct answer is 7(8 – 1)(6).
5. Here, you are asked to write your own formula once again. However, this time it is for a geometric sequence. The first term is 25, and you must also find the common ratio. To get from 25 to –5, you must divide by –5. This also works to get from –5 to 1, so the common ratio is –1/5. Your formula should look like this:

$$a_n = 25\left(-\frac{1}{5}\right)^{n-1}$$

Systems of Equations

1. When solving systems of equations, the best thing to do first is to isolate one of the variables. In this problem, you can do so by changing the sign on the bottom equation:

$$x - 2y = 14$$
$$-x + 4y = 8$$

Add the two equations together:

$$2y = 22$$
$$y = 11$$

Choose one of the original equations and substitute 11 for y. Solve for x.

$$x - 2(11) = 14$$
$$x - 22 = 14$$
$$x = 36$$

It is always a good idea to test your answers by substituting x and y values into both of the original equations.

2. This problem is a little trickier than the first, as you cannot simply change the sign of one of the equations to isolate one of the variables. In this situation, you have to make the coefficients the same through multiplication. Since you know that 4 and 6 both go into 12, use the x term. Multiply the top equation by 3, and the bottom by 2:

$$12x - 6y = 18$$
$$-12x + 10y = 14$$

Add the two equations together:

$$4y = 32$$
$$y = 8$$

Finally, choose one of the original equations, substitute 8 for y, and solve for x.

$4x - 2(8) = 6$
$4x - 16 = 6$
$4x = 22$
$x = \frac{22}{4}$, or $\frac{11}{2}$

3. The first step is rearranging the equations to align like terms:

$3x - y = 18$
$4x + 6y = 24$

Multiply the top equation by 6 and add the equations:

$18x - 6y = 108$
$+4x + 6y = 24$
$22x = 132$
$x = 6$

Now choose one of the original equations, substitute 6 in for x, and solve for y:

$3(6) - y = 18$
$18 - y = 18$
$-y = 0$
$y = 0$

4. First, distribute the 8 through the parentheses to get $8y + 8x = 12$. You can then multiply the second equation by -2 to isolate one of the variables, and rearrange the equations to line up the like terms:

$8x + 8y = 12$
$-8x + 6y = 44$

Add the equations together:

$14y = 56$
$y = 4$

Now choose one of the original equations, substitute 4 for y, and solve for x.

$8\ (x + 4) = 12$
$8x + 8(4) = 12$
$8x + 32 = 12$
$8x = -20$
$x = -\frac{20}{8}$
$x = -\frac{5}{2}$

5. First, line up the like terms in both equations:

$4x - y = 63$
$x + 3y = 6$

Multiply the top equation by 3 and add the equations.

$12x - 3y = 189$
$x + 3y = 6$
$13x = 195$
$x = 15$

Now substitute 15 for x in one of the equations.

$x+3y=6$
$15+3y=6$
$3y=-9$
$y=-3$

Logarithms

1. $\log_x 27=3$ means the log to the base x of $27=3$. This means that x^3 must equal 27, and therefore x must equal 3.
2. By definition, $\log_a b=c$, if $a^c=b$. In this question, you are asked to find the value of a. You are given the values of b and c, so your equation should look like this:
   $x^4=625$
   You need to find a number that, when raised to the 4th power, equals 625. Test the answer choices: $4^4=256$, $7^4=2401$, $5^4=625$. Therefore, the correct answer is 5. You could immediately eliminate 7 after finding that 7^4 is already substantially larger than 625.
3. To solve, turn the logarithm into an equation with an exponent:
   $3^x=729$
   Test some values for x:
   $3^2=9$
   $3^3=27$
   $3^4=81$
   $3^5=243$
   $3^6=729$
   Therefore, log 3 (729) = 6.
4. By definition, $\log_x 196=2$ means the log to the base x of $196=2$. This means that x^2 must equal 196. To find the answer, you can simply take the square root of 196, which is 14.
5. By definition, if $\log_7 x=3$, then $7^3=x$. Therefore, $x=343$.

Roots of Polynomials

1. To solve this problem by factoring, you can start with a $2x$ on one side, and an x on the other:
   $(2x +/- \_\_)(x +/- \_\_)$
   These two missing numbers must add up to 9 (keep in mind that one of them is being multiplied by 2), and also must multiply to give −35. The only possible factors of 35 are 1, 5, 7, and 35. In looking at the problem, 5 and 7 seem like the most logical choices. You can try a few different combinations, but you should come up with:
   $(2x-5)(x+7)$
   To find the roots, set each quantity equal to 0:
   $2x-5=0,\ x+7=0$
   $2x=5,\ x=-7$
   $x=\frac{5}{2}$ and $x=-7$

2. To solve this problem, begin with an x in both factors:

   $(x +/- \_)(x +/- \_)$

   The two missing numbers must have a sum of 2 and a product of −3. 3 is only divisible by 1 and 3, and the sum must be 2, so 3 is positive and 1 is negative.

   $(x-1)(x+3)$

   $x-1=0, x+3=0$

   $x=1$ and $x=-3$

3. For this problem, you will have to work backward; you are already given the roots, and are being asked to find the equation to which they belong. Since the roots given are 6 and −2, you can write out $x-6=0$ and $x+2=0$. Now, to find the original equation, you must multiply these two quantities:

   $(x-6)(x+2)$
   $x^2-6x+2x-12$
   $x^2-4x-12$

4. To solve this problem, start with x in each of the factors:

   $(x +/- \_)(x +/- \_)$

   The sum of the missing numbers must be −8, and the product must be 16. Therefore, the numbers must both be −4.

   $(x-4)(x-4)$

   This can also be written $(x-4)^2$. Solve for x:

   $x-4=0$

   $x=4$

5. Since the a value is −1, start with x and $-x$ in the factors.

   $(x +/- \_)(-x +/- \_)$

   The sum must be 3 and the product must be 40, but remember that for the sum, one of the numbers is being multiplied by −1. In this case, 8 and 5 are the correct values:

   $(x+5)(-x+8)$
   $x+5=0$ and $-x+8=0$
   $x=-5$ and $x=8$

Number Line Graphs

1. The answer is 8. Distance is always positive and can be shown as absolute value: $|-5-3|=8$. You can also draw a number line, label −5 and 3, and see that the distance between those two points is 8.
2. The midpoint is simply the point that is exactly halfway between the two points given. It can be thought of as an average. This value can be determined using the following formula:

   $$\text{Midpoint}=\frac{1}{2}(x_1+x_2)$$

   $$\text{Midpoint}=\frac{1}{2}(-3+2)$$

   $$\text{Midpoint}=\frac{1}{2}(-1)$$

   $$\text{Midpoint}=-\frac{1}{2}$$

3. The answer is $x \geq -3$ AND $x < 6$. AND is used because the bold line is connecting the two points. If there were a space, OR would be used. This eliminates the third choice. Open circles signify > or < and closed circles signify ≥ or ≤. This eliminates the first and second choices.
4. First, determine the values of x. Since both circles are open, > and < are used. Also, there is a space between the two points, so OR will be used. In the end, you have $x < 2$ OR $x > 6$. Now it is simply a matter of substituting the x values into the equations and determining which one is correct. The third choice, $|2x - 10| > 6$, is the correct answer.
5. There is a space between the two points, so use OR. This eliminates the second and fourth answer choices. The third choice is incorrect because the graph does not show a bold line for values greater than −6. Also, the open circle means < or > needs to be used, as the values do not include −6. The first choice, $x \geq 2$ OR $x < -6$, is correct.

Equation of a Line and Slope of a Line

1. First, rearrange the equation into the slope-intercept form by isolating y. In this case, you divide by 2:

   $y = 2x + 6$

   In the slope-intercept formula, $y = mx + b$, b is the y-intercept. Because $b = 6$, the y-intercept is (0, 6).
2. Rearrange the equation into the slope-intercept form by isolating y. In this case, you divide by 3:

   $y = -\frac{2}{3}x + 5$

   You know that in the slope-intercept formula, $y = mx + b$, m is the slope. Because $m = -\frac{2}{3}$, the correct answer is $-\frac{2}{3}$.
3. This equation represents a vertical line; the y-intercept is 0, so the line is parallel to the y-axis. A vertical line has an undefined slope. This is because slope is equivalent to "rise over run." If the "run" is 0, the slope must be undefined because 0 cannot divide into anything.
4. Remember that in the slope-intercept form, $y = mx + b, m$ is the slope and b is the y-intercept. In addition, parallel lines have the same slope; therefore, the slope of both lines (m) is 4. You are given that the y-intercept (the point at which the line crosses the y-axis) is 3. The equation of the line will be $y = 4x + 3$.
5. First, rearrange the equation into slope-intercept form, by subtracting $3x$ and $-y$ from both sides:

   $y = -3x + 2$

   For two lines to be perpendicular, their slopes must be negative reciprocals. The negative reciprocal of −3 is $\frac{1}{3}$ The problem also states that the perpendicular line has a y-intercept of 8. If you substitute $\frac{1}{3}$ for m and 8 for b in the slope-intercept equation, you get $y = \frac{1}{3}x + 8$.

Distance and Midpoint Formulas

1. The distance formula is: Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   You can substitute the given values of x and y into the formula to solve for the distance, as follows:

   $$\text{Distance} = \sqrt{(3-9)^2 + (-4-4)^2}$$

$\text{Distance} = \sqrt{(6)^2 + (8)^2}$
$\text{Distance} = \sqrt{36 + 64}$
$\text{Distance} = \sqrt{100}$
$\text{Distance} = 10$

2. You can use the distance formula [$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$] to solve this problem:

   $17 = \sqrt{(-6-2)^2 + (y-8)^2}$

   Square both sides.

   $289 = (-8)^2 + (y-8)^2$

   $289 = (64) + (y-8)^2$

   $225 = (y-8)^2$

   Take the square root of both sides.

   $15 = y - 8$

   $23 = y$

   The following equation is also correct:

   $17 = \sqrt{(2-(-6))^2 + (8-y)^2}$

   Square both sides.

   $289 = (2+6)^2 + (8-y)$
   $289 = 64 + (8-y)$
   $225 = (8-y)$

   Take the square root of both sides.

   $15 = (8-y)$
   $7 = -y$
   $-7 = y$

3. Use the midpoint equation to solve this problem. First solve for the x-coordinate, which is half the distance between 12 and 10:

   $x_m = \frac{(x_2 + x_1)}{2}$

   $x_m = \frac{(10+12)}{2}$

   $x_m = \frac{22}{2}$

   $x_m = 11$

   Do the same for y_m, which is half the distance between 5 and −7:

   $y_m = \frac{(y_2 + y_1)}{2}$

   $y_m = \frac{(5 + -7)}{2}$

   $y_m = -\frac{2}{2}$

   $y_m = -1$

   Therefore, the midpoint is (11, −1).

4. You only have to solve for the x-coordinate because you are given the y-coordinate:

$$x_m = \frac{(x_2 + x_1)}{2}$$
$$5 = \frac{(-2 + x_1)}{2}$$
$$10 = -2 + x_1$$
$$12 = x_1$$

5. Use the distance formula: Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to solve this problem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{Distance} = \sqrt{(5 - 0)^2 + (0 - 5)^2}$$
$$\text{Distance} = \sqrt{(5)^2 + (-5)^2}$$
$$\text{Distance} = \sqrt{(25 + 25)}$$
$$\text{Distance} = \sqrt{50}$$
$$\text{Distance} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}.$$

Properties and Relations of Plane Figures

1. The area of a triangle and the length of one of the legs of a right triangle are given. However, you need the length of both legs to use the Pythagorean Theorem to determine the hypotenuse. Since you have the area, start there. For a right triangle, Area $= \frac{1}{2}$ (base) $\times$ (height). You are given the base and area, so solve for the height:

$$54 = \frac{1}{2}(9) \times (\text{height})$$
$$54 = 4.5\ (\text{height})$$
$$12 = \text{height}$$

Now you know the lengths of the two legs of the right triangle and can use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to calculate the hypotenuse:

$$9^2 + 12^2 = c^2$$
$$81 + 144 = c^2$$
$$225 = c^2$$
$15 = c$. The hypotenuse is 15 cm.

2. The formula for the area of a circle is: area $= \pi r^2$. The formula for the circumference of a circle is: $C = 2\pi r$. Since you are given the circumference, you can use that to find the radius, r, and then use the radius to find the area:

$$14\pi = 2\pi r$$
$$14 = 2r$$
$$r = 7$$

Now substitute r into the equation for area:

$$\text{Area} = \pi(7^2)$$
Area $= 49\pi$. The area of the circle is 49π in^2.

3. A parallelogram's angles add up to 360°: $360° - 35° = 325°$.

4. The equation for the area of a trapezoid is: area $= \frac{1}{2}(\text{base}_1 + \text{base}_2)$ (height). Substitute the given variables into the equation and solve for the missing base:

$$30 = \frac{1}{2}(8 + \text{base}_2)(3)$$

$$10 = \frac{1}{2}(8 + \text{base}_2)$$

$$20 = (8 + \text{base}_2)$$

$$\text{base}_2 = 12 \text{ ft}$$

5. A square is a special kind of rectangle. All of its sides are equal in length. Since the area of a rectangle is area $= l \times w$, the area of a square would be area $= s^2$ (side squared) because length and width are equal. For this problem, the given side is 6 mm. If the figure were a square, the area would be 36 mm^2. However, the area is said to be 42 mm^2. Therefore the shape is a rectangle and not a square.

Angles, Parallel Lines, and Perpendicular Lines

1. Supplementary angles add together to total 180°. Therefore, the supplementary angle to a 40° angle is a 140° angle.

2. Supplementary angles add together to total 180°. Therefore the supplementary angle to a 25° angle is a 155° angle.

3. The 90° angle marked indicates that the other three angles formed by the intersection of lines p and o each measure 90° also. As line n is parallel to line m, the same four 90° angles are formed at the intersection of lines p and m. Similarly, the angles on line o each measure 90°, too, because lines p and o are parallel. Thus, angle $\theta = 90°$.

4. The transversal crosses two parallel lines, so the angles made at the intersections will be identical. 43° corresponds to the supplementary angle of a on line y. Since 43° and a are supplementary angles, they must add up to 180°. Therefore, the answer is $180° - 43° = 137°$.

5. Since line v is perpendicular to line t, it forms four right angles. The line segment that is unnamed in the diagram dissects one of the right angles. Angle a is one side and 35° is the measurement given for the other side. These two angles add up to 90°: $90° - 35° = 55°$. Therefore, the angle measures 55°.

Perimeter, Area, and Volume

1. The question asks you to determine the number of bags of fertilizer that will cover your rectangular backyard. According to information in the problem, 6 pounds of fertilizer can cover 700 square feet. Begin by calculating the area of the rectangular backyard. The area of a rectangle is determined by multiplying the length (70 feet) by the width (40 feet):

$$70 \times 40 = 2{,}800.$$

The area of the rectangular backyard is 2,800 square feet. The problem states that 6 pounds of fertilizer can cover 700 square feet. Calculate the number of times that 700 will go into 2,800:

$$2{,}800 \div 700 = 4.$$

You will need 4 times 6 pounds of fertilizer to treat 2,800 square feet:

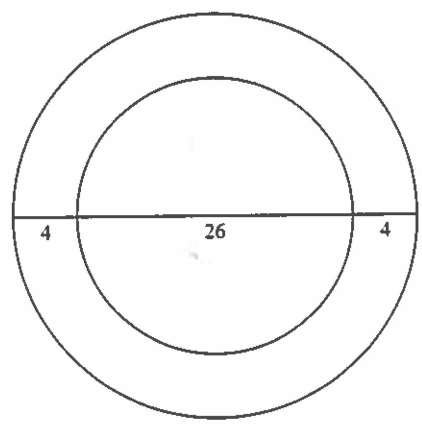
$$4 \times 6 = 24.$$

Since you will need a total of 24 pounds of fertilizer to treat the backyard, and each bag of fertilizer weighs 8 pounds, divide 24 by 8 to find the number of bags of fertilizer you will need:

$$24 \div 8 = 3.$$

You will need 3 bags of fertilizer to treat a backyard that measures 2,800 square feet.

2. If the pool has a diameter of 26 feet, and the fence needs to be 4 feet away from the edge of the pool, the diameter of the area enclosed by the fence would be $26+4+4=34$ feet. Draw a picture to help visualize the problem:



The area of a circle is πr^2. The radius is half of the diameter, so $r = 17$. Substitute 17 for r and 3.14 for π and solve:

$\text{Area} = (3.14)\ (17)^2$

$\text{Area} = 907.46\ \text{ft}^2$

3. A beach ball is a sphere, and the formula for the volume of a sphere is: $\left(\frac{4}{3}\right)\pi r^3$. The diameter is given as 0.6 m, so the radius is half of that, 0.3 m. Substitute that value into the formula and compute the volume:

$$\text{Volume} = \left(\frac{4}{3}\right)\pi(0.3^3)$$

$$\text{Volume} = \left(\frac{4}{3}\right)\pi(0.027)$$

$\text{Volume} = 0.036\pi\ \text{m}^3$, or approximately $0.113\ \text{m}^3$

4. The formula for the volume of a cylinder is $\pi r^2 h$. The question is asking for diameter, so first solve for r, then double it.

$$350 = \pi r^2\left(\frac{14}{\pi}\right)$$

$r^2 = 25$ cm
$r = 5$ cm
Since the radius is 5 cm, the diameter is 10 cm.

5. The equation for the volume of a cube is: s^3. Since we are given an edge, or side (s) of 5, you simply substitute 5 for s. The answer is 125 in^3.

Trigonometry

1. Using the mnemonic SOHCAHTOA helps you remember that sine is the ratio of "opposite to hypotenuse." The side opposite of a has a length of 12. The hypotenuse has a length of 13. So, $\sin a = \frac{12}{13}$.
2. Using the mnemonic SOHCAHTOA helps you remember that tangent is the ratio of "opposite to adjacent" and cosine is "adjacent over hypotenuse." Since you are given cosine, you know the lengths of two sides of the right triangle. The adjacent leg is 4 and the hypotenuse is 5. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$), you can calculate the length of the opposite leg, and then calculate tan a:

   $$a^2 + 4^2 = 5^2$$
   $$a^2 + 16 = 25$$
   $$a^2 = 9$$
   $$a = 3$$

   Now you have the adjacent (4) and opposite (3) legs, so $\tan a = \frac{3}{4}$.
3. By definition, to convert degrees to radians multiply by $\frac{\pi}{180}$:

   $$\frac{60\pi}{180} = \frac{\pi}{3}$$
4. To convert radians to degrees $\frac{180}{\pi}$:

   $$\frac{3\pi}{4} = \left(\frac{180}{\pi}\right)$$
   $$= \frac{540\pi}{4\pi}$$
   $$= 135$$
5. By definition, secant is the reciprocal of cosine, which is calculated by dividing the length of the adjacent side by the length of the hypotenuse (adj/hyp). Therefore, $\cos a = 5/13$, and the length of the side adjacent to the angle is 5, while the length of the hypotenuse is 13. By definition, sine is equivalent to opposite/hypotenuse, so you must use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to find the length of the side opposite angle a:

   $$a^2 + 5^2 = 13^2$$
   $$a^2 + 25 = 169$$
   $$a^2 = 144$$
   $$a = 12$$

   Because $a = 12$, the sin of angle $a = \frac{12}{13}$.

Translating Word Problems

1. You are given that Tom started out with 6 books. After he gave 2 books to his sister he was left with $6 - 2$ books. He then purchased 3 more books, so he now has $6 - 2 + 3$ books.
2. To solve this problem, start with William and work backward. William walked 2 miles, and Rebecca walked 4 times as far as William. Therefore, Rebecca walked 4(2) miles. Juan walked 3 more miles than Rebecca, so Juan walked $4(2) + 3$ miles.
3. The first step is to calculate the total cost of the CDs: $2(15) = 30$. You are given that, in addition to the 2 CDs, Tina also purchases d of the DVDs, each of which costs \$18. Therefore, her cost for the DVDs was $18d$. Now simply add the terms together to get $30 + 18d$.
4. You are given that Mark, m, is older than Frank, f. Therefore, $f < m$. You are also given that Mark, m, is younger than David, d. Therefore, $m < d$. Mark's age is between Frank and David's ages, so $f < m < d$.
5. You are given that Jim is j years old today; therefore, 2 years ago, Jim would have been $j - 2$ years old. At that time, Kathy was twice as old as Jim, or $2(j - 2)$.