

COMPLEX NUMBERS

From:

$$x^2 + 1 = 0$$

$$x^2 + 1 + (-1) = 0 + (-1)$$

$$x^2 = (-1)$$

WE GET:

$$x = \sqrt{(-1)}$$

WE CALL $\sqrt{(-1)} \rightarrow i$ (A COMPLEX UNIT)

$$i^2 = (-1)$$

SO:

$$i = \sqrt{(-1)}$$

$$i^2 = (-1)$$

$$i^3 = (-i)$$

$$i^4 = 1$$

A COMPLEX NUMBER: $a + bi$

↑ REAL PART ↑ $\sqrt{(-1)}$ IMAGINARY PART

WE TREAT i AS ANY OTHER NUMBER FOLLOWING THE RULES OF ALGEBRA.

OPERATIONS

- ADDITION AND SUBTRACTION

$$1) i + 6i = 7i$$

$$2) (2 + 3i) + (-1 + 2i)$$

$$\underline{2} + 3i + \underline{(-1)} + 2i$$

$$1 + \underline{3i} + \underline{2i}$$

$$\boxed{1 + 5i}$$

$$3) (-6 + 4i) - (-8 + 5i)$$

$$\underline{(-6)} + \underline{4i} + \underline{8} + \underline{(-5i)}$$

$$\boxed{2 + (-i) \text{ OR } 2 - i}$$

- MULTIPLICATION

$$4) i(i) = i^2 = (-1)$$

$$5) 3i(4i) = 12i^2 = 12(-1) = (-12)$$

$$6) (2 - 3i)(-3 + 4i)$$

$$\underline{2} + \underline{(-3i)}$$

$$\underline{(-3)} + \underline{4i}$$

$$\underline{8i} + \underline{(-12i^2)}$$

$$\underline{(-6)} + \underline{9i}$$

$$\underline{(-6)} + \underline{17i} + \underline{(-12)(-1)}$$

$$\underline{(-6)} + \underline{17i} + \underline{12}$$

$$6 + 17i$$

$$\boxed{6 + 17i}$$

7) (3-i)(3+i)

$$\begin{array}{r} 3+(-i) \\ 3+i \\ \hline 3i+(-i^2) \\ 9+(-3i) \\ \hline 9+(-i^2) \quad (-i^2) = -(-1) = 1 \\ 9+1 \\ \hline 10 \end{array}$$

(3-i)(3+i) = 10

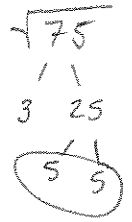
3-i and 3+i are called conjugate pairs

multiply conjugate pairs → GENERAL RULE: (a+bi)(a-bi) GIVES A POSITIVE REAL NUMBER = a²+b²

ALSO NOTE: √-75 = √75 √(-1)

$$\sqrt{-75} = \sqrt{75} i$$

$$\sqrt{-75} = 5\sqrt{3} i$$



- i⁰ = 1
- i¹ = i
- i² = -1
- i³ = -i
- i⁴ = 1

FINDING THE VALUE OF iⁿ THE CYCLE OF THE POWERS OF i REPEAT FOR EVERY FOURTH POWER.

According to the remainder upon dividing the exponent n by 4 gives the new power, let's see an example

- 1) i⁹ 9 ÷ 4 = 2 WITH A REMAINDER OF 1
i⁹ = i
- 2) i¹⁸ 18 ÷ 4 = 4 WITH A REMAINDER OF 2
i¹⁸ = (-1)
- 3) i³⁵ 35 ÷ 4 = 8 WITH A REMAINDER OF 3
i³⁵ = -i
- 4) i⁸⁰ 80 ÷ 4 = 20 WITH A REMAINDER OF 0
i⁸⁰ = 1

CW21 - Complex Numbers Operations

Simplify.

1) $7 - (5i) + (-5 - 6i)$

2) $(-6 + 3i) + (6 - i)$

3) $(5i) - (3i) - (-4 + 7i)$

4) $(1 - 2i) + (-6 - 5i)$

5) $(i)(-4i)(-3 - 5i)$

6) $(i)(2i)(8 - 8i)$

7) $(4 - 3i)(-1 - 7i)$

8) $(i)(5i)(4 + 8i)$

9) $(-3i)(1 - 3i)(6 - 3i)$

10) $(8i)(4 + 5i)(-1 + 8i)$

11) $(-7 + 8i)(8 + 2i)(-6 - 6i)$

12) $-4(2 + 2i)(-8 - 3i)$

13) $(-1 + 2i) - (2i) - (8 - 6i)$

14) $(5 - 8i) - (6 - 4i) + (5i)$

15) $(2 - 6i) + (5 - 2i) - (2 - 4i)$

16) $(6 - i) - (3 + i) + (-4 - i)$

Answers to CW21 - Complex Numbers Operations

1) $2 - 11i$

2) $2i$

3) $4 - 5i$

4) $-5 - 7i$

5) $-12 - 20i$

6) $-16 + 16i$

7) $-25 - 25i$

8) $-20 - 40i$

9) $-63 + 9i$

10) $-216 - 352i$

11) $732 + 132i$

12) $40 + 88i$

13) $-9 + 6i$

14) $-1 + i$

15) $5 - 4i$

16) $-1 - 3i$

HW21 - Complex Numbers Operations

Period _____

Simplify.

1) $(5 - i) - (6i) - (i)$

2) $(-7 - 8i) - (5 + 6i)$

3) $(2 + 7i) + (4 - 4i)$

4) $(6i) - (-1 - 7i) + (4i)$

5) $-4 + (7 - 2i) - (8i)$

6) $(-7 + 5i) - (7 + 8i)$

7) $(6 - 8i) + (-8 + 8i)$

8) $(8 - 8i) - (5 - 7i)$

9) $(-4 - 4i) - (6i) + (3i)$

10) $(6i) + 6 - (8 + 2i)$

11) $(-4 + 8i)^2$

12) $(3 + 5i)(-1 - 2i)$

13) $(7 - 7i)^2$

14) $(5 + 2i)(-3 + 4i)$

15) $(-2i)(-3i)(1 + 3i)$

16) $(1 + 3i)^2$

17) $(6 + 5i)^2$

18) $(6 - 3i)(-5 + 2i)$

19) $(-7 + 7i)(8 + 4i)$

20) $(-5 + 3i)(-4 + 3i)$

21) $(7 - 5i)(-4 + 5i)(-4 + 4i)$

22) $(-8 + i)(-2 + 8i)(-8 + i)$

23) $(-8i)(-2 - 7i)(-1 - 2i)$

24) $2(8 - 6i)(8 + 4i)$

25) $(4i)(-7 - 2i)(-5 - 4i)$

26) $-6(-3 + 8i)(4 - 7i)$

27) $(-6i)(4 - 3i)(2 - 8i)$

28) $(-2i)(4 + i)(8 - 7i)$

29) $-6(6 - 6i)(-2 + 5i)$

30) $(7i)(6 - 4i)(-8 - i)$

31) $(-4 + 7i) - (7 + 8i) + (-1 - 7i)$

32) $5 - (5 + 5i) - (6 - 5i)$

33) $(7 - 8i) - (-6 + 4i) - (-4 - 2i)$

34) $(-4 - 8i) - (1 + 6i) + (-4 - 4i)$

35) $(-4 + 8i) - (-7 + 4i) - (6 - 5i)$

36) $(4 + 2i) - (6 - 2i) + (3i)$

37) $(-8 + 2i) - (6 - 2i) - (-2 - 3i)$

38) $(3 - 2i) + (2 + 3i) + (i)$

39) $(5 + 2i) + (-5 - 6i) - (7i)$

40) $(7i) - (-3 + 7i) - (-5 + i)$

Answers to HW21 - Complex Numbers Operations

1) $5 - 8i$

5) $3 - 10i$

9) $-4 - 7i$

13) $-98i$

17) $11 + 60i$

21) $-208 - 232i$

25) $-152 + 108i$

29) $-108 - 252i$

33) $17 - 10i$

37) $-12 + 7i$

2) $-12 - 14i$

6) $-14 - 3i$

10) $-2 + 4i$

14) $-23 + 14i$

18) $-24 + 27i$

22) $2 + 536i$

26) $-264 - 318i$

30) $-182 - 364i$

34) $-9 - 18i$

38) $5 + 2i$

3) $6 + 3i$

7) -2

11) $-48 - 64i$

15) $-6 - 18i$

19) $-84 + 28i$

23) $88 + 96i$

27) $-228 + 96i$

31) $-12 - 8i$

35) $-3 + 9i$

39) $-11i$

4) $1 + 17i$

8) $3 - i$

12) $7 - 11i$

16) $-8 + 6i$

20) $11 - 27i$

24) $176 - 32i$

28) $-40 - 78i$

32) -6

36) $-2 + 7i$

40) $8 - i$